Numerical Linear Algebra Assignment 16

Exercise 1. (TreBau Exercise 37.1, 10 points)

The standard recurrence relation for Legendre polynomials is

$$P_n(x) = \frac{2n-1}{n} x P_{n-1}(x) - \frac{n-1}{n} P_{n-2}(x) \tag{1}$$

with initial values $P_0(x) = 1$, $P_1(x) = x$.

(a) Compute $P_2(x)$ and $P_3(x)$.

(b) Since $\{P_n(x)\}$ and $\{q_{n+1}(x)\}$ are normalized differently, (1) is not the same as the recurrence

$$x q_n(x) = \beta_{n-1} q_{n-1}(x) + \alpha_n q_n(x) + \beta_n q_{n+1}(x)$$

with coefficients

$$\alpha_n = 0, \qquad \beta_n = \frac{1}{2}(1-(2n)^{-2})^{-1/2}.$$

Write down the two tridiagonal matrices corresponding to these formulas, and derive the relationship between them.

(c) Use the result of (b) to determine a formula for $q_{n+1}(1)$, or equivalently, for $\|P_n\|$.

Exercise 2. (Programming, TreBau Exercise 37.4, 10 points)

(a) Write a six-line Matlab program that computes the nodes and weights for the n-point Gauss–Legendre quadrature formula and applies these numbers to compute the approximate integral of the function f.

(b) Taking $f(x) = \mathrm{e}^x$ and $n = 4$, confirm the example in the text. Then plot $|I(\mathrm{e}^x) - I_n(\mathrm{e}^x)|$ on a log scale for $n = 1, 2, \ldots, 40$ and comment on the results.

(c) Produce a similar plot for $f(x) = \mathrm{e}^{|x|}$, and comment.